# Nuclear Physics

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# Outlines

- Some properties of nucleus
- Binding energy
- Nuclear magnetic moment
- Nuclear Models

# Mass and Charge of Nucleons

**Atom = Nucleus + Orbital electrons**

**Nucleus = Protons + Neutrons**

**Symbol:** ${}^{A}_{Z}\mathrm{X}$;
$Z$= No. of protons
$A$= No. of nucleons (protons+neutrons)=mass number
X = Chemical symbol of the element

**Proton**
**Mass** ($m_P$) = $1.67262 \times 10^{-27} Kg$
**Charge** = $+e$ = $1.602 \times 10^{-19} C$

**Neutron**
**Mass** ($m_N$) = $1.67493 \times 10^{-27} Kg$;
**Charge = 0**

# Nuclear size and atomic mass unit (a.m.u.)

**Nuclear Size**

**Radius** $(R) \sim 10 - 100fm$; $1fm = 10^{-15}m$

$R = r_0 A^{\frac{1}{3}}$, **where** $r_0$ **is called nuclear radius parameter.**

$1a.m.u = \frac{1}{16} \times$ **mass of an** $^{16}O$ **atom.**

$1u = \frac{1}{12} \times$ **mass of a** $^{12}C$ **atom** $= 1.6605 \times 10^{-27} Kg = 931.5 MeV/c^2$

Table: *Rest mass in Kgs, Unified atomic mass units (u) and* $MeV/c^2$

| | | Mass | |
|---|---|---|---|
| Object | Kg | u | $MeV/c^2$ |
| Electron | $9.1014 \times 10^{-31}$ | 0.00054858 | 0.511 |
| Proton | $1.67262 \times 10^{-27}$ | 1.007276 | 938.27 |
| $^1_1$H atom | $1.7353 \times 10^{-27}$ | 1.007825 | 938.78 |
| Neutron | $1.67493 \times 10^{-27}$ | 1.00865 | 939.57 |

# Isotopes, Isobars and Isotones

- **Isotopes:** The nuclei of an element which have different mass numbers, i.e. same Z but different A.
- Ex.($^{1}_{1}$H, $^{2}_{1}$H, $^{3}_{1}$H), ($^{35}_{17}$Cl, $^{37}_{17}$Cl)
- **Isotopic Abundances:** The radio of masses of (or relative percentages) different isotopes present in an element collected from natural sources is caled isotopic abdunces.
- In Natural Hydrogen $^{1}_{1}$H remains 99.99% and $^{2}_{1}$H remains 0.01%.

- **Isobars:** Nuclei which have same mass number (A) but different proton number (Z).
- Ex. ($^{40}$S,$^{40}$Cl, $^{40}$Ar, $^{40}$Ca), ($^{58}$Fe, $^{58}$Ni)

- **Isotones:** he Nuclei which have the same neutron number but different proton number, i.e. same N but different Z.
- Ex. $^{16}_{8}$O, $^{15}_{7}$N, $^{14}_{6}$C; in each cane $N$=8.

# Nuclear mass defect and binding energy

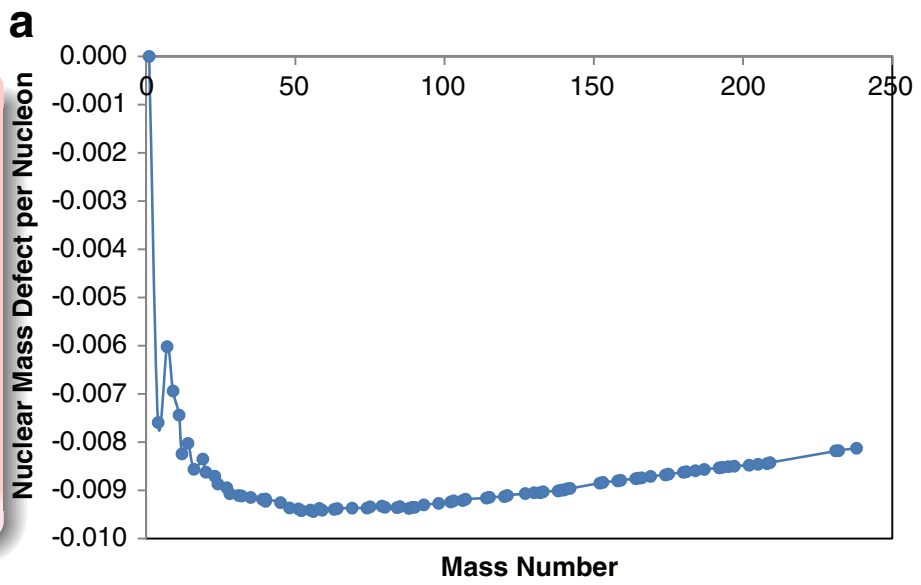
The deviation of atomic mass (in g) from the actual mass number is called **nuclear mass defect**.

$\Delta M = M(^A_Z\mathrm{X}) - A$

Nuclear mass defect per nucleon($f$):

$f = \frac{\Delta M}{A} = \frac{M(^A_Z\mathrm{X}) - A}{A} = \frac{M(^A_Z\mathrm{X})}{A} - 1$

$\therefore M(^A_Z\mathrm{X}) = (1 + f)A$

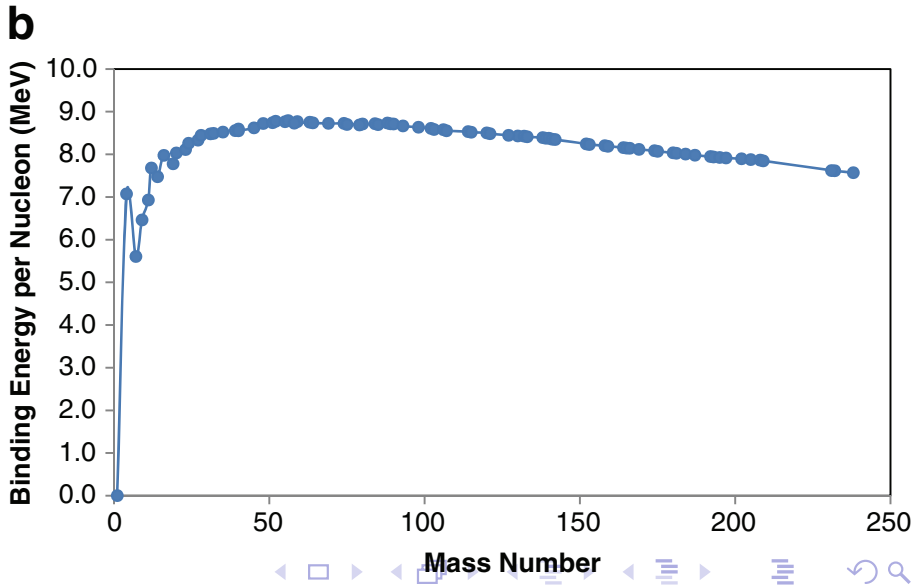



The energy required to separate the nucleons out of a nuclueus is called the **nuclear binding energy** of the nucleus.

$B.E. = Z(m_P) + N(m_N) - M(^A_Z\mathrm{X})$

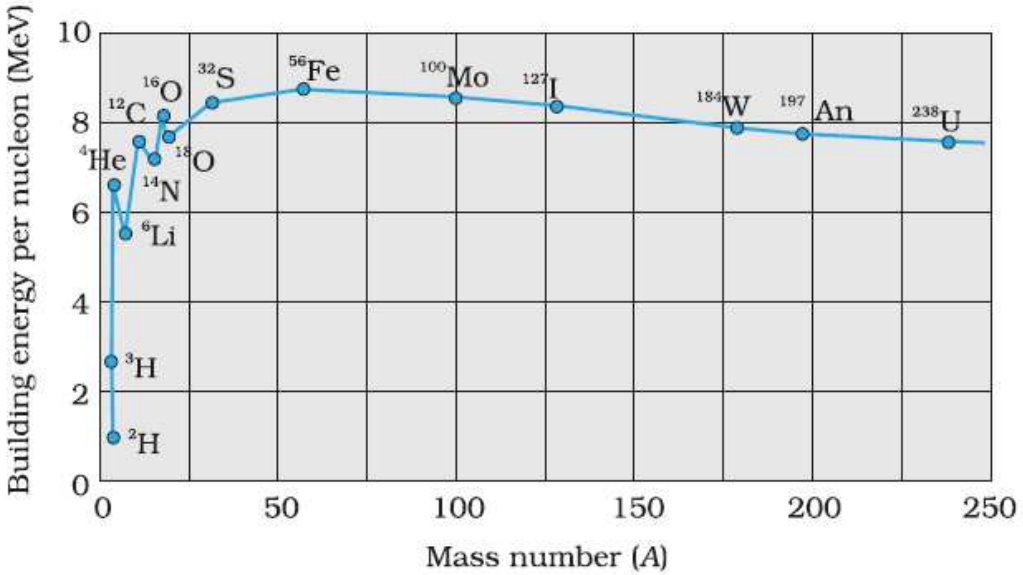
Binding energy per nucleon or binding fraction($f_B$):

$f_B = \frac{B.E.}{A} = \frac{Z(m_P) + N(m_N) - M(^A_Z\mathrm{X})}{A}$

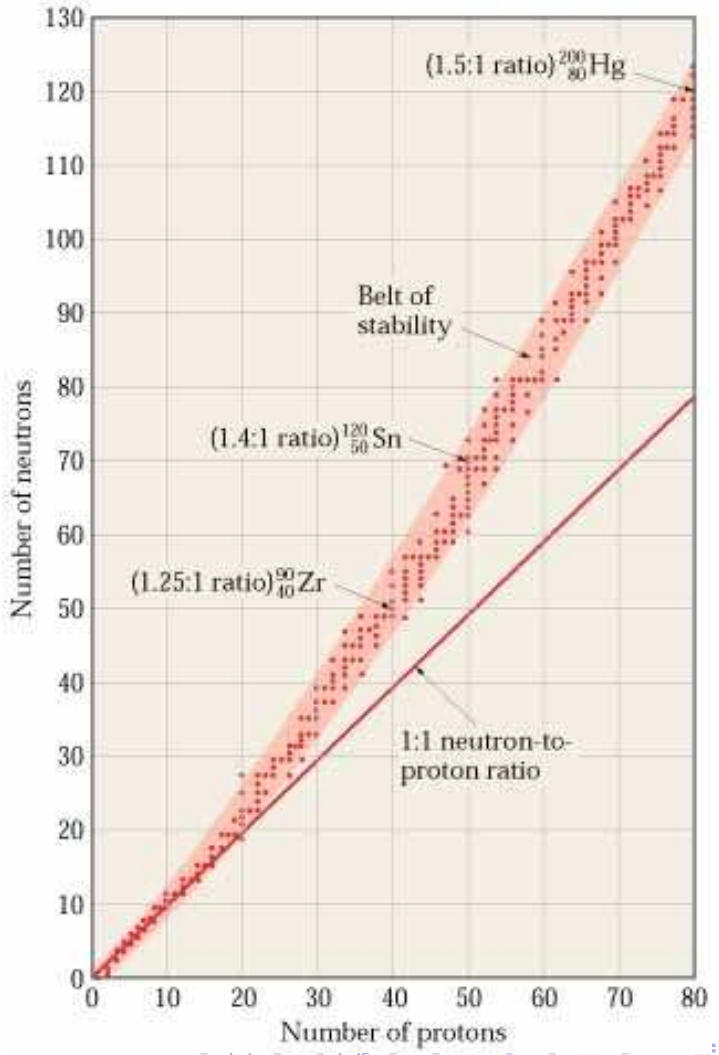


# Variation of binding energy with mass number

**The higher the binding energy per nucleon, the more stable is the nucleus.**




The binding energy per nucleon as a function of mass number.



# Magnetic moments of nucleons

**Both the protons and neutrons have magnetic dipole moments, although neutron is charge less particle.**

$\mu_P = 2.79285\mu_N$ **and** $\mu_n = -1.913\mu_N$
**where,** $\mu_N = \frac{e\hbar}{2m_P}$**, is called the Nuclear magnetron.**

**Both proton and neutron obay FD statistics, so they are spin-**$\frac{1}{2}$ **particles.**

# Density distribution of nuclei

Considering the nucleus to be a sphere of uniform density, we can write,

$$\rho = \frac{Am_n}{\frac{4\pi}{3}R^3} \tag{1}$$

$$\implies R = r_0 A^{\frac{1}{3}} \tag{2}$$

where, $r_0$ is a constant, called the nuclear radius parameter. $r_0 \sim 1.1 - 1.3fm$. Since the material of the nucleus is uniformly distributed, so we can calculate the mean square radius of the nucleus.

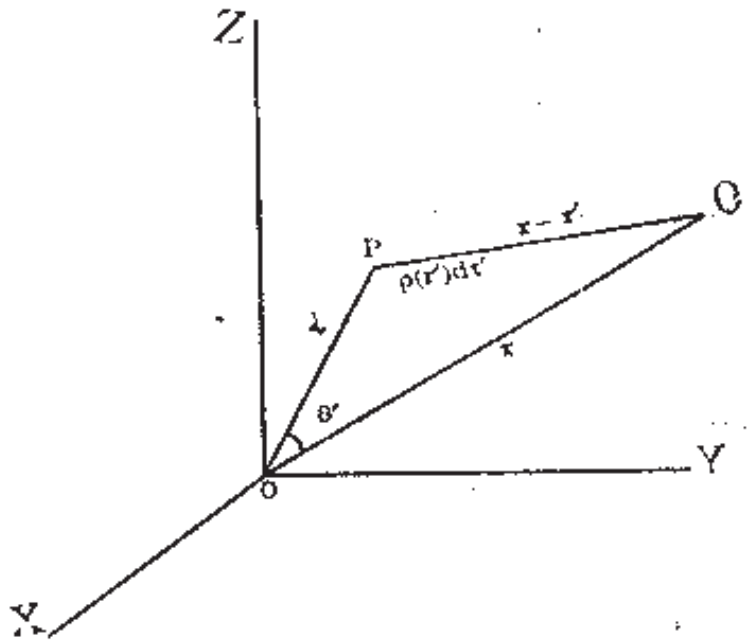
$$\langle r^2 \rangle = \frac{\int_0^R r^2 4\pi r^2 \rho dr}{\int_0^R 4\pi r^2 \rho dr} \tag{3}$$

$$= \frac{3}{5}r^2 \tag{4}$$

$$\therefore R^2 = \frac{5}{3}\langle r^2 \rangle \tag{5}$$

# Electric multipole moments of nuclei

Nuclei are actually not spherical but spheroidal. Let, P be a point inside a nucleus and O be the origine of a co-ordinate system and $\vec{OP} = \vec{r'}$. If the charge densiy at P be $\rho(\vec{r'})$, then the charge for an infinitisimal volume elememt $d\tau'$ is $\rho(\vec{r'})d\tau'$




The electric potential due to this charge at a point Q ($\vec{OQ} = \vec{r}$, say) will be,

$$dV(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\rho(\vec{r'})d\tau'}{|\vec{r} - \vec{r'}|} \tag{6}$$

Let, the angle between $\vec{r}$ and $\vec{r'}$ be $\theta'$. Then,

$$|\vec{r} - \vec{r'}|^{-1} = (r^2 - 2rr'cos\theta' + r'^2)^{-\frac{1}{2}} \tag{7}$$

$$\implies |\vec{r}-\vec{r'}|^{-1} = \frac{1}{r}\left[1-\left(\frac{2r'}{r}\right)cos\theta'+\left(\frac{r'}{r}\right)^2\right]^{-\frac{1}{2}}$$

$$= \frac{1}{r}\left[1+\left(\frac{r'}{r}\right)cos\theta'+\left(\frac{r'}{r}\right)^2\frac{1}{2}\left(3cos^2\theta'-1\right)+.....\right] \quad (8)$$

We know the Legendre polynomials are given by,

$$P_0(cos\theta') = 1$$
$$P_1(cos\theta') = cos\theta'$$
$$P_2(cos\theta') = \frac{1}{2}\left(3cos^2\theta'-1\right)$$

etc.

Now using eq(8) in eq(6), we get the potential as,

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \rho(\vec{r'})\frac{1}{r}\left[1+\left(\frac{r'}{r}\right)cos\theta'+\left(\frac{r'}{r}\right)^2\frac{1}{2}\left(3cos^2\theta'-1\right)+.....\right]dr' \quad (9)$$

Let the point Q be placed on the $Z$-axis, then $rcos\theta' = z'$ (say); so the above relation can be expressed as,

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{e}{r}\left[Z + \frac{D}{r} + \frac{1}{2}\frac{Q}{r^2} + ....\right] \tag{10}$$

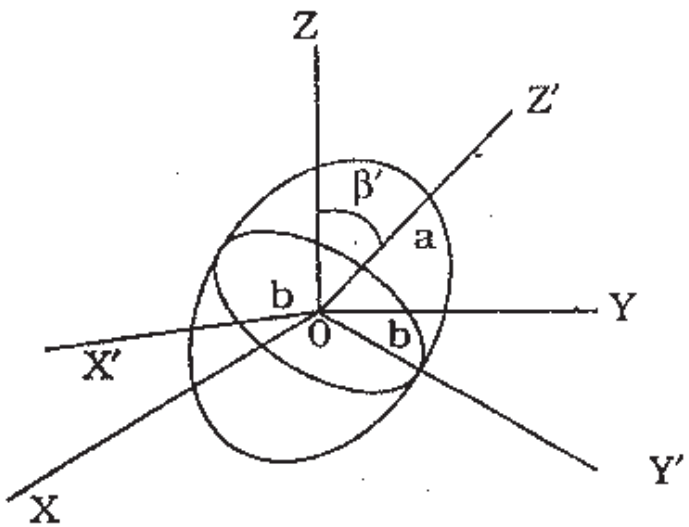
where,

$$\begin{aligned} Z &= \frac{1}{e}\int \rho(\vec{r'})d\tau' \\ D &= \frac{1}{e}\int \rho(\vec{r'})z'd\tau' \\ Q &= \frac{1}{e}\int \rho(\vec{r'})(3z'^2 - r'^2)d\tau' \end{aligned}$$

Here, $Z$ is the number of protons in the nucleus, and $D$ and $Q$ are the dipole and quadrupole moments of the nucleus respectively. $D$ and $Q$ have the dimensions of length and area respectively.

# Geometrical significance of electric quadrupole moment of nuclei

Let's consider the nucleus as a uniformly charged spheroid. Here $X, Y, Z$ are the space fixed coordinate ases and $X', Y', Z'$ are the nuclear fixed coordinate axes. Let $a, b$ and $b$ are the radii of the spheroid along $Z', X'$ and $Y'$ axes respectively and $\beta'$ is the angle between $Z$ and $Z'$ axes.




In nuclear fixed axis, the equation of the spheroid is,

$$\frac{x'2}{b^2} + \frac{y'^2}{b^2} + \frac{z'^2}{a^2} = 1 \tag{11}$$

and the charge density of the nucleus is,

$$\rho = \frac{Ze}{\frac{4\pi}{3}ab^2} \tag{12}$$

where, $Z$ is the number of protons in the nuceus.

Taking the cylinderical coordinate system $(s', z', \phi')$, eq(11) resuces to,

$$\frac{s'^2}{b^2} + \frac{z'^2}{a^2} = 1 \tag{13}$$

where, $s'^2 = x'^2 + y'^2$. So, from eq(13), we have,

$$s'^2 = b^2\left(1 - \frac{z'^2}{a^2}\right) \tag{14}$$

Therefore, in cylinderical coordinate the expression for quadrupole moment ($Q$) becomes (taking $\rho$ fixed),

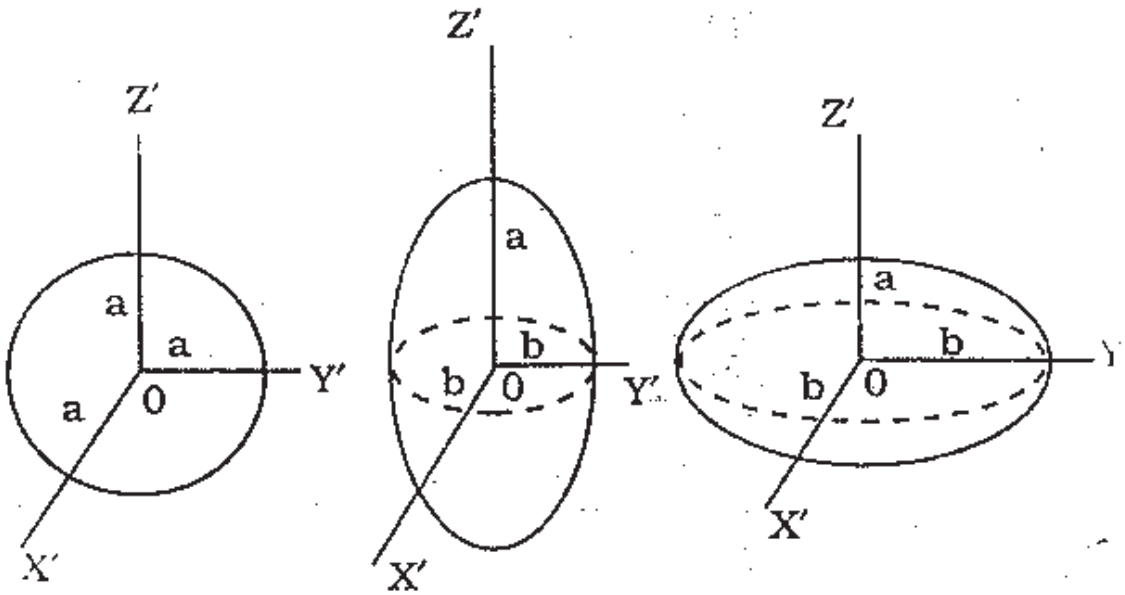
$$Q = \frac{\rho}{e}\int_{-a}^{a} dz' \int_{0}^{(b^2 - z'^2/a^2)} (2z'^2 - s'^2)s'\,ds' \int_{0}^{2\pi} d\phi' \tag{15}$$

Now putting the value of $\rho$ from eq(12) into eq(15), we have,

$$Q = \frac{2}{5}Z(a^2 - b^2) \tag{16}$$

# Geometrical significance of electric quadrupole moment of nuclei

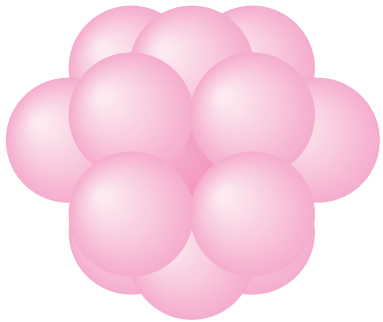
1. If $b = a$, then $Q = 0$; i.e., when the value of quadrupole moment of a nucleus is zero, then the nucleus will be spherical.
2. If $b < a$, then $Q > 0$; i.e., when the value of $Q$ is positive, then the nucleus will be a prolate spheroid.
3. If $b > a$, then $Q < 0$; i.e., when $Q$ is negative then the nucleus will be oblate spheroid.

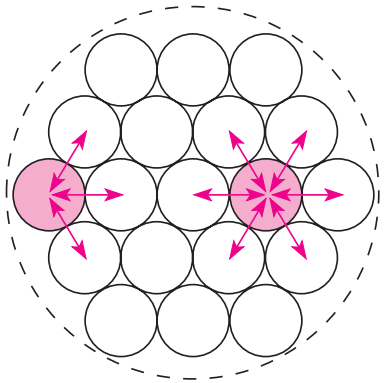


# Nuclear Model: Liquid drop model

Assumptions:

- Nuclues is considered to be a spherica liquid drop consisting of a number of smallest derplets.
- Each nucleon interacts only with its nearest neighbors.
- The bond energy between two nucleons is assumbed to be $U$, which is shared by two nucleons, so each has binding energy $\frac{1}{2}U$. Attractive energy is taken to be positve and repulsive to be negative.





- **Volume energy:** In closed packed structure, each interior sphere has 12 nearest neighbors, so each interior nucleon has binding energy of $12(\frac{1}{2}U)$ or $6U$. If all $A$ (mass number) nucleons were in its interior, the total binding energy of the nuclues would be

$$E_v = 6AU \tag{17}$$

  It may also be written as,

$$E_v = a_1 A \tag{18}$$

- **Surface energy:** The surface nucleons have less number of nucleons($< 12$). A nucleus of radius $R$ has an surface area of $4\pi R^2 = 4\pi R_0^2 A^{\frac{2}{3}}$. So,

$$E_s = -a_2 A^{\frac{2}{3}} \tag{19}$$

# Nuclear Model: Liquid drop model

- **Coulomb repulsion energy:** The potential energy of a pair of protons $r$ apart is equal to

$$V = -\frac{e}{4\pi\epsilon_0 r} \tag{20}$$

Since there are ${}^{Z}C_2 = \frac{Z(Z-1)}{2}$ pairs of protons,

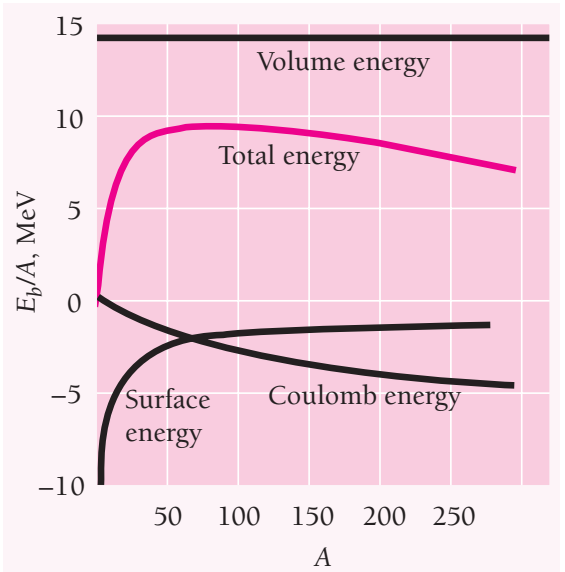
$$E_c = \frac{Z(Z-1)}{2} V = -\frac{Z(Z-1)e^2}{8\pi\epsilon_0} \left(\frac{1}{r}\right)_{av} \tag{21}$$

where, $\left(\frac{1}{r}\right)_{av}$ is the value of $\left(\frac{1}{r}\right)$ averaged over all proton pairs. For uniform distribution of protons in the nucleus $\left(\frac{1}{r}\right)_{av} \propto \frac{1}{R} \propto \frac{1}{A^{\frac{1}{3}}}$, so,

$$E_c = -a_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}} \tag{22}$$

**Therefore the total binding energy of the nucleus is,**

$$E_b = E_v + E_s + E_c = a_1 A - a_2 A^{\frac{2}{3}} - a_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}} \tag{23}$$

**Therefore, binding energy per nucleon is,**

$$\frac{E_b}{A} = a_1 - a_2 A^{-\frac{1}{3}} - a_3 \frac{Z(Z-1)}{A^{\frac{4}{3}}} \tag{24}$$