Susceptibility is more or less a measure of how susceptible (or sensitive) the material is to a magnetic field.

putting eq (7) in eq (6) we get

$$\vec{B} = \mu_0 (1 + \chi_m) \vec{H} = \mu \vec{H} \qquad (8)$$

or $$\vec{B} = \mu_0 \mu_r \vec{H} \qquad (9)$$

where $$\mu_r = 1 + \chi_m = \frac{\mu}{\mu_0} \qquad (10)$$

8. Magnetic Boundary Conditions:

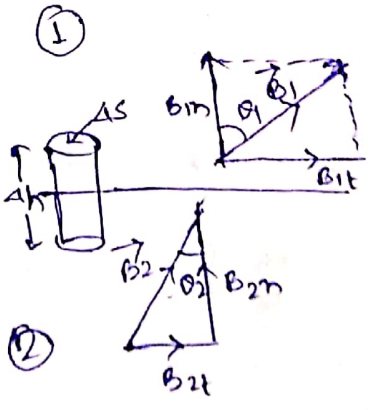
The conditions, which must be satisfied by the fields $\vec{B}$ (or $\vec{H}$) at the boundary between two different media, are called magnetic boundary conditions.

From magnetostatics, we know

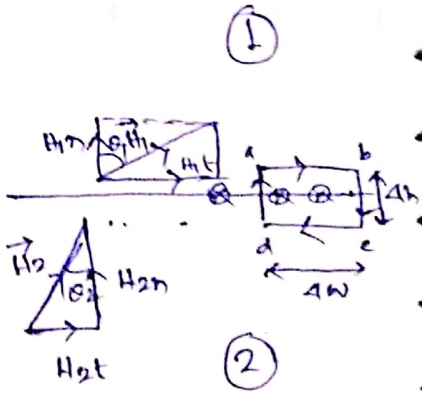
$$\oint \vec{B} \cdot d\vec{s} = 0 \qquad (1)$$

and $$\oint \vec{H} \cdot d\vec{l} = I_{free} \qquad (2)$$

Consider the boundary between two magnetic media (1) and (2), ~~check~~ characterized respectively, by μ_1 and μ_2 (Fig (a) and (b)).



(a)



(b)

Applying eq (1) to the pillbox of figure and letting $\Delta h \to 0$, we get

$$B_{1n} \Delta S - B_{2n} \Delta S = 0 \qquad (3)$$

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or $B_{1n} = B_{2n}$

or $\mu H_{1n} = \mu_2 H_{2n}$ (4)

Similarly, we apply eq (2) to the closed path 'abcda' of fig (b) where surface current K_f on the boundary is assumed normal to the path. we obtain,

$$H_{1t}\,\Delta w + H_{1n}\frac{\Delta h}{2} + H_{2n}\frac{\Delta h}{2} - H_{2t}.\Delta w - H_{2n}\frac{\Delta h}{2} - H_{1n}\frac{\Delta h}{2} = K_f \Delta w \quad (5)$$

As $\Delta h \to 0$, eq (5) gives

$$H_{1t} - H_{2t} = K_f \quad (6)$$

$$\Rightarrow \frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K_f \quad (7)$$

If the fields make an angle θ with the normal to the interface,

eq (4) becomes

$$B_1 \cos\theta_1 = B_2 \cos\theta_2 \quad \left[\because B_{1n} = B_1\cos\theta_1,\ B_{2n} = B_2\cos\theta_2\right] \quad (8)$$

and (7) becomes,

$$\frac{B_1}{\mu_1}\sin\theta_1 - \frac{B_2}{\mu_2}\sin\theta_2 = K_f \quad (9)$$

and when the surface does not contain free current ($K_f = 0$), so

$$\frac{B_1}{\mu_1}\sin\theta_1 = \frac{B_2}{\mu_2}\sin\theta_2 \quad (10)$$

Dividing eq (10) by eq (8),

$$\frac{1}{\mu_1}\tan\theta_1 = \frac{1}{\mu_2}\tan\theta_2$$

or $$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}} \quad (11)$$

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Inductors and inductances :

A circuit carrying current I produces a magnetic field $\vec{B}$ which causes a flux $\psi = \int \vec{B} \cdot d\vec{s}$ to pass through each turn of the circuit as shown in fig 1.

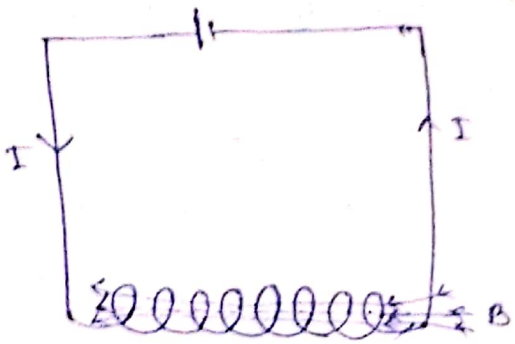

fig 1.

If the circuit has N identical turns, we define the flux linkage Φ as

$$\Phi = N\psi \tag{1}$$

If the medium surrounding the circuit is linear, the flux linkage Φ is proportional to the current I producing it, i.e.

$$\Phi \propto I$$

or

$$\Phi = LI \tag{2}$$

where L is a constant of proportionality, called the inductance of the circuit.

From eq (1) and (2), we have

$$L = \frac{\Phi}{I} = \frac{N\psi}{I} \tag{3}$$

The unit of inductance is Henry (H) which is same as Wb/A.

The inductance defined in eq(3) is commonly known as self-inductance since the linkages are produced by the inductor itself.

- Energy stored in an inductor:

The magnetic energy stored in an inductor is

$$W_m = \frac{1}{2} LI^2 \tag{4}$$

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Mutual Inductance:

When we have two circuits carrying currents I_1 and I_2 as shown in fig 2, a magnetic interaction exists between the circuits.

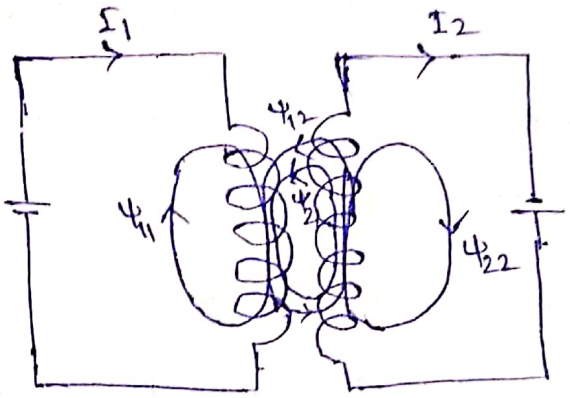

Fig 2.

Four component fluxes ψ_{11}, ψ_{12}, ψ_{21} and ψ_{22} are produced.

ψ_{12} = Flux passing through circuit 1 due to current I_2 in circuit 2.

If $\vec{B}_2$ is the magnetic field due to I_2 and S_1 is the area of the circuit 1, then

$$\psi_{12} = \int_{S_1} \vec{B}_2 \cdot d\vec{S} \qquad (5)$$

We define the mutual inductance M_{12} as the ratio of flux linkage $\Phi_{12} = N_1 \psi_{12}$ on circuit 1 to current I_2, that is

$$M_{12} = \frac{\Phi_{12}}{I_2} = \frac{N_1 \psi_{12}}{I_2} \qquad (6)$$

Similarly, the mutual inductance M_{21} is defined as the flux linkage of circuit 2 per unit current I_1, i.e.

$$M_{21} = \frac{\Phi_{21}}{I_1} = \frac{N_2 \psi_{21}}{I_1} \qquad (7)$$

It can be shown by using energy concepts that if the medium surrounding the circuit is linear (i.e. in the absence of ferromagnetic material).

$$M_{12} = M_{21} \qquad (8)$$

We define the self inductance of circuits 1 and 2, respectively, as

$$L_1 = \frac{\Phi_{11}}{I_1} = \frac{N_1 \psi_1}{I_1} \quad \text{and} \quad L_2 = \frac{\Phi_{22}}{I_2} = \frac{N_2 \psi_2}{I_2} \qquad (9)$$

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Faraday's law:

Faraday discovered that the induced emf, V_{emf} (in volts) in any closed circuit is equal to the time rate of change of magnetic flux linkage by the circuit.

This is called Faraday's law, and it can be expressed as

$$V_{emf} = -\frac{d\Phi}{dt} = -N\frac{d\psi}{dt} \qquad (1)$$

where N is the number of turns and ψ is the flux through each turn. The negative sign shows that the induced voltage acts in such a way to oppose the flux ~~produced~~ producing it. This is known as Lenz's law.

Transformer and motional emfs:

Considering the connection between emf and electric field, we may examine how Faraday's law links electric and magnetic fields.

$$V_{emf} = -\frac{d\Phi}{dt} \qquad (2)$$

In terms of $\vec{E}$ and $\vec{B}$, (2) can be written as

$$V_{emf} = \oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S} \qquad (3)$$

The variation of flux with time may be caused in three ways

(i) By having a stationary loop in a time varying $\vec{B}$ field.

(ii) By having a time varying loop area in a static $\vec{B}$ field.

(iii) By having a time varying loop area in a time varying $\vec{B}$ field.

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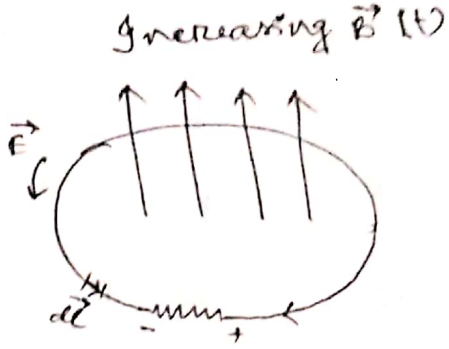
A. Stationary loop in a time varying $\vec{B}$ field:

$$V_{emf} = \oint \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$



by applying stokes theorem

$$\int_S (\nabla \times \vec{E}) \cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\Rightarrow \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{[Differential form of Faradays law]}$$

B. Moving loop in static $\vec{B}$ field (Motional emf):

When a conducting loop is moving in a static $\vec{B}$ field, an emf is induced in the loop. From the Lorentz force law, the force on a charge moving with uniform velocity $\vec{u}$ in a magnetic field $\vec{B}$ is

$$\vec{F_m} = Q\vec{u} \times \vec{B} \qquad (1)$$

We define the motional electric field $\vec{E_m}$ as

$$\vec{E_m} = \frac{\vec{F_m}}{Q} = \vec{u} \times \vec{B}. \qquad (2)$$

If we consider a conducting loop moving with uniform velocity $\vec{u}$ as consisting of a large number of free electrons, the emf induced in the loop is

$$V_{emf} = \oint_L \vec{E_m} \cdot d\vec{l} = \oint_L (\vec{u} \times \vec{B}) \cdot d\vec{l} \qquad (3)$$

This type of emf is called motional emf because it is due to the motional action.

When $\vec{B}$ and $\vec{u}$ are perpendicular, we have

$$\vec{F_m} = I\vec{l} \times \vec{B} \quad \text{or} \quad |\vec{F_m}| = IlB. \qquad (4)$$

and $V_{emf} = ulB. \qquad (5)$

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c. Moving loop in time varying field:

This is the general case in which a moving conducting loop is in a time varying magnetic field. Both transformer emf and motional emf are present.

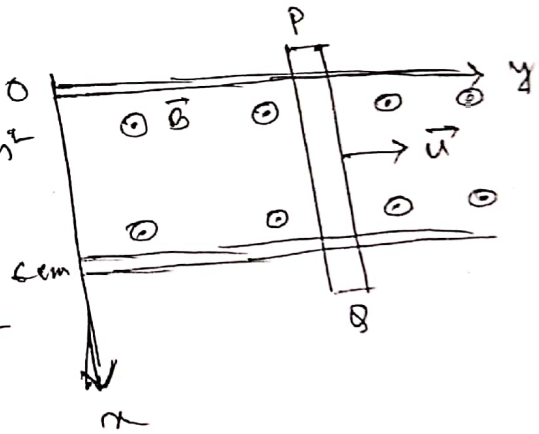
$$\text{Vemf} = \oint_L \vec{E}.d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{s} + \oint_L (\vec{u}\times\vec{B}).d\vec{l}$$

$$\text{or,} \quad \int_S (\nabla\times\vec{E}).d\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{s} + \int_S \{\nabla\times(\vec{u}\times\vec{B})\}.d\vec{s}$$

Applying stokes theorem on L.H.S. and 2nd term of R.H.S.

$$\therefore \quad \nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} + \nabla\times(\vec{u}\times\vec{B}).$$

Ex.1. A conducting bar can slide freely over two conducting rails as shown in fig. calculate the induced voltage in the bar.



(a) If the bar is stationed at $y = 8$ cm and $\vec{B} = 4\cos 10^4 t\ \hat{z}$ mWb/m^2

(b) If the bar slides at a velocity $\vec{u} = 20\hat{y}$ m/s and $\vec{B} = 4\hat{z}$ mWb/m^2

(c) If the bar slides at a velocity $\vec{u} = 20\hat{y}$ m/s and $\vec{B} = 4\cos(10^4 t - y)\hat{z}$ mWb/m^2.

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