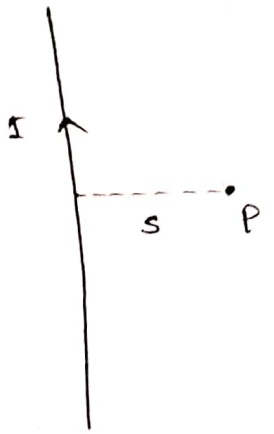
1. Curl of $\vec{B}$ :




The magnetic field due to a straight current carrying conductor at a distance s is given, in terms of cylindrical polar coordinates $(s, \phi, z)$, by

$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi} \qquad (1)$$

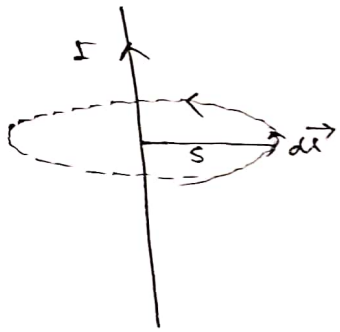
The length element in cylindrical coordinates is

$d\vec{l} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$, so line integral of $\vec{B}$ is

$$\oint \vec{B}.d\vec{l} = \frac{\mu_0 I}{2\pi}\oint \frac{1}{s}\,s\,d\phi = \frac{\mu_0 I}{2\pi}\oint d\phi = \mu_0 I \cdot (2)$$

$$\therefore \boxed{\oint \vec{B}.d\vec{l} = \mu_0 I_{enc}} \quad -(3)$$

[ $I_{enc}$ is the current enclosed by the loop ]




If the flow of charge is represented by the volume current density $\vec{J}$, then

$$I_{enc} = \int_S \vec{J}.d\vec{a} \qquad (4)$$

where the integral is taken over the surface bounded by the loop

Applying stokes' theorem to eq (4), we have,

$$\oint \vec{B}.d\vec{l} = \mu_0 I_{enc}$$

$$\Rightarrow \int_S (\nabla \times \vec{B}).d\vec{a} = \mu_0 \int_S \vec{J}.d\vec{a}$$

$$\Rightarrow \boxed{\nabla \times \vec{B} = \mu_0 \vec{J}} \quad \text{[ For any finite surface ]}$$

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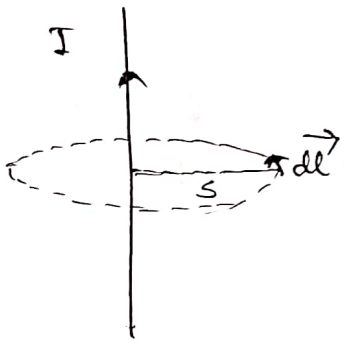
## 2. Ampere's law and its applications :

. The line integral of magnetic field over a closed loop is equal to the $\mu_0$ times the current enclosed by the loop.

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}} \quad \text{(Integral form)}$$

or $$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}} \quad \text{(Differential form)}$$

Ex1. Find the magnetic field at a distance s from a long straight wire, carrying a steady current I :



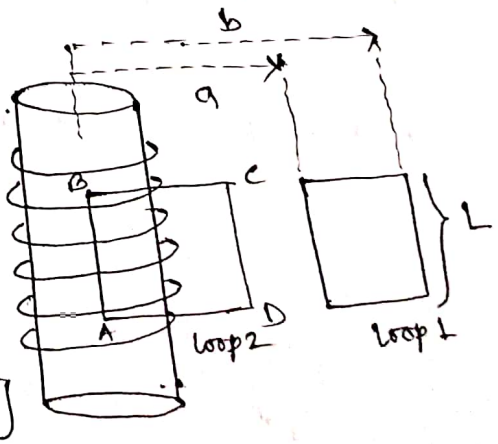

By symmetry, $\vec{B}$ is constant around an Amperian loop of radius s, centered on the wire (As shown in the fig). So, Ampere's law gives

$$\oint \vec{B} \cdot d\vec{l} = B \oint dl = B\, 2\pi s = \mu_0 I_{enc} = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi s}$$

Ex2. Find the magnetic field of a very long solenoid, consisting of closely wound turns per unit length on a cylinder of radius R and carrying a steady current I.

Ans.




Since loop1 does not enclose any current,

so $\oint_{loop1} \vec{B} \cdot d\vec{l} = 0$

$\Rightarrow B_{out} = 0$ [$B_{out}$ is the magnetic field outside the solenoid]

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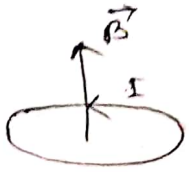
In Loop 2




$$\oint_{\text{Loop2}} \vec{B}.d\vec{l} = \mu_0 (nL) I$$

$$\Rightarrow \int_{AB} \vec{B}.d\vec{l} + \int_{BC} \vec{B}.d\vec{l} + \int_{CD} \vec{B}.d\vec{l} + \int_{DA} \vec{B}.d\vec{l} = \mu_0 (nL) I$$

since magnetic field is perpendicular to the circular current loops, therefore the ~~integral (B) and 4~~ 2nd and 4th integral on the L.H.S. vanishes. the part CD remains outside the solenoid, hence ~~CD~~ 3rd integral also vanishes. so, from the 1st integral, we have

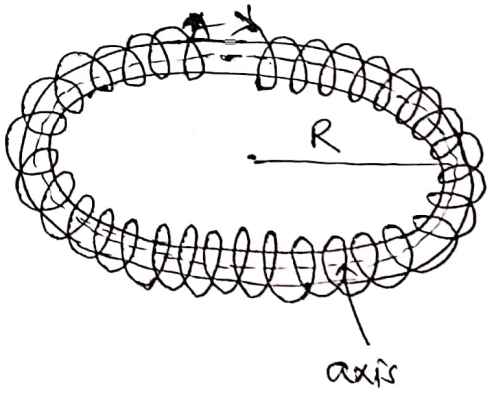
$$BL = \mu_0 (nL) I$$

$$\Rightarrow B = \mu_0 n I$$

or $\vec{B} = \mu_0 n I \hat{z}$

Ex3. Find the magnetic field of a toroid along its axis having N number of turns on it and the amount of current flowing is I.

If the radius of the toroid is R then the length of its circumference is $2\pi R$. Considering the toroid as a solenoid of length $2\pi R$, the magnetic field ~~at~~ along its axis is given by




$$B.(2\pi R) = \mu_0 NI$$

$$\Rightarrow B = \frac{\mu_0 NI}{2\pi R}$$

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