True or False Answers

https://www.amazon.in/Joseph-A-Gallian/e/B001ILIAMK/ref=dp_byline_cont_pop_book_1

Theorems mentioned below are from the book of **Joseph-A-Gallian**

Serial Number	Answer
1.	The smallest prime is 2
2.	It is true if a and b are relatively prime
3.	a = b or a = -b
4.	See the corollary to Theorem 0.2
5.	lcm(10,12) = 60
6.	See Exercise 5 in Chapter 0
7.	Theorems 0.5 and 0.6
8.	D_n has n rotations and n reflections
9.	A reflection change orientation from clockwise to counter clockwise
10.	A reflection changes orientation from clockwise to counter clockwise. So odd number of reflections reverses orientation
11.	By definition, the identity is the only element of order 1
12.	(ab)^{-1} = b^{-1}a^{-1}. See Theorem 2.4
13.	The concept of taking roots is not defined for groups in general
14.	Instead of dividing by an element a, multiply by the inverse of a
15.	It is true if and only if n is prime. See Example 20
16.	63 is in the group U(100)
17.	Not all such matrices have inverses with integer entries
18.	All such matrices have inverses with integer entries
19.	This is true if and only if ab = ba. Compare with Exercise 34 of Chapter 2
20.	It is true for Abelian groups
21.	The union need not be closed
22.	See Exercise 32 of Chapter 3 in the 8th ed.; Ex. 18 in the 7th
23.	The definition permits this to be true
24.	See Theorem 3.3
25.	Every group with more than one element contains at least two subgroups
26.	See Exercise 48 of Chapter 3 of the 8th ed; Ex. 13 of the 7th
27.	See Exercise 52 of Chapter 3 of the 8th ed; Ex. 36 of the 7th
28.	|D_n| = 2n
29.	Z(D_n) = {R_0, R_180} when n is even; Z(D_n) = {R_0} when n is odd
30.	The operations are different

31.	All the elements of the group commute with all the elements of the group
32.	Let G be a non-Abelian group and a in the center of G. Then C(a) = G
33.	Must also know that H is nonempty
34.	Z_3 does not have the same operation as Z_6
35.	Z_n has order n
36.	See Theorem 4.3
37.	Note that powers of a commute with powers of a
38.	See Theorem 3.4
39.	If g is a generator so is its inverse
40.	If $\|a\| = 10$, then $\|a^{10}\| =1$, $\|a^5\| =2$, $\|a^2\| = 5$
41.	D_3 is a counterexample
42.	Take $a = b^{-1}$. Or in Z_6, take a = 1 and b = 2
43.	$\|g\|$ divides n. See Corollary 2 of Theorem 4.1
44.	If $m = \text{lcm}(\|a\|,\|b\|)$, then $(ab)^m = e$
45.	Look at the real numbers 1 and -1
46.	The rotation of smallest positive degree is a generator
47.	See Theorem 4.4
48.	See the Corollary of Theorem 4.4
49.	See the Corollary of Theorem 4.4
50.	The statement is true for finite groups (see the Corollary of Theorem 4.4)
51.	See Theorem 4.3
52.	The statement is true when m and n are relatively prime
53.	The statement is true if d divides the order of the group
54.	This is the corollary to Theorem 4.4
55.	Chap 5 (123) and (12) do not commute and they belong to S_n for all $n >= 3$
56.	This statement is true when the cycles are disjoint
57.	The evenness or oddness of a product of permutations depends only number of permutations in the product that are odd permutations. Since only an even number of odd permutations is an even permutation n must be even
58.	See Theorem 6.1 (Cayley's Theorem)
59.	By definition
60.	To be true the set must be finite. See Exercise 12 in Chapter 5 of the 8th Ex. 10 in the 7th
61.	
62.	See Example 2 of Chapter 6

<table>
<tr><td>63.</td><td>See Example 2 of Chapter 6</td></tr>
<tr><td>64.</td><td>See Example 2 of Chapter 6</td></tr>
<tr><td>65.</td><td>The set of rotations in D_n is a cyclic group of order n</td></tr>
<tr><td>66.</td><td>See Exercise 1 of Chapter 6</td></tr>
<tr><td>67.</td><td>See Exercise 6 of Chapter 6</td></tr>
<tr><td>68.</td><td>See Example 5 in Chapter 7. This is true for cyclic groups</td></tr>
<tr><td>69.</td><td>This is a corollary to Lagrange's Theorem</td></tr>
<tr><td>70.</td><td>Both are equal to |H|</td></tr>
<tr><td>71.</td><td>The statement is true if the cosets are both right cosets or both left cosets</td></tr>
<tr><td>72.</td><td>See part 9 of the lemma about properties of cosets</td></tr>
<tr><td>73.</td><td>The correct condition b^{-1}a is in H</td></tr>
<tr><td>74.</td><td>See Corollary 1 of Theorem 7.1</td></tr>
<tr><td>75.</td><td>This is Therorem 7.3</td></tr>
<tr><td>76.</td><td>This is Corollary 1 of Therorem 7.1</td></tr>
<tr><td>77.</td><td>It is true when G is finite</td></tr>
<tr><td>78.</td><td>xA_n = yA_n if and only if y^{-1}x belongs to A_n, which is true if and only if x and y are both even or both odd</td></tr>
<tr><td>79.</td><td>This follows from the Orbit-Stablizer Theorem</td></tr>
<tr><td>80.</td><td>True if H and K are finite</td></tr>
<tr><td>81.</td><td>See Theorem 7.3</td></tr>
<tr><td>82.</td><td>This is true when s and t are relatively prime</td></tr>
<tr><td>83.</td><td>See Theorem 8.2</td></tr>
<tr><td>84.</td><td>The two groups do not have the same order</td></tr>
<tr><td>85.</td><td>This is true when s and t are relatively prime</td></tr>
<tr><td>86.</td><td>This follows from the definition</td></tr>
<tr><td>87.</td><td>This is true in the case that the elements have finite order</td></tr>
<tr><td>88.</td><td>This is true if and only if |a| and |b| are relatively prime</td></tr>
<tr><td>89.</td><td>The subgroup is <m/r> + <n/s></td></tr>
<tr><td>90.</td><td>The Z_5 + Z_3 is isomorphic Z_3 + Z_5 which isomorphic to a subgroup of Z_9 + Z_25</td></tr>
<tr><td>91.</td><td>See Threorem 8.3</td></tr>
<tr><td>92.</td><td>The statement is true if p is odd</td></tr>
<tr><td>93.</td><td>See Example 3 of Chapter 8</td></tr>
<tr><td>94.</td><td>|U(4)| = 16; |(U(4) + U(10)| = 8</td></tr>
<tr><td>95.</td><td>See Theorem 8.3. 12 and 25 are relatively prime</td></tr>
<tr><td>96.</td><td>False. H must be normal</td></tr>
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97.	False. ah = h'a for some h' in H
98.	False. {e} and G are normal subgroups of G
99.	False. If H is not normal in G then aHbH = abH is not a binary operation on the cosets
100.	True. See Exercise 12 of Chapter 9 of the 8th ed.; Ex. 10 in the 7th
101.	True. See Exercise 13 of Chapter 9 in 8th ed.; Ex 1 in the 7th
102.	False. Observe that $|S_n/A_n| = 2$
103.	True. See Exercise 9 of Chapter 9 in the 8ed.; Ex. 7 in the 7th
104.	False. Exercise 55 of Chapter 9 in 8th ed.; Ex. 53 in the 7th
105.	False. a^n belongs to N
106.	False. $|a|$ must be finite for this to be true
107.	False. $|a|$ must be finite for this to be true
108.	False.$|a|$ must be finite for this to be true
109.	False. The statement is true if H or K is normal
110.	True. The statement is true if H or K is normal
111.	True. This is the corollary of Theorem 9.7
112.	True. See Exercise 58
113.	True. See Theorem 9.7
114.	True. See Theorem 9.6
115.	False. Must also have the intersection of HK and L is the identity
116.	
117.	A one-to-one homomorphism is an isomorphism onto the image
118.	The mapping that takes every element to the identity is a homomorphism
119.	See part 2 of Theorem 10.2
120.	See part 3 of Theorem 10.2
121.	See Example 11 of Chapter 10
122.	The statement is true when $|a|$ is finite
123.	f(H) is normal in f(G)
124.	See Part 6 of Theorem 10.2
125.	The statement is true if G is finite
126.	See Theorem 10.3
127.	See Theorem 10.3
128.	See the corollary to Theorem 11.1
129.	The set of even integers under addition does not have a unity
130.	See Exercise 2 of Chapter 12
131.	See the paragraph following the definition of a field

132.	See Exercise 22 of Chapter 12
133.	See Theorem 13.3
134.	See the paragraph following the definition of characteristic
135.	Units can be cancelled
136.	In Z_6, 3^2 =3
137.	See Exercise 15 of Chapter 14
138.	This need not be true when R does not have a unity
139.	The ring must have a unity for this to be true
140.	The ring must have a unity for this to be true
141.	See Corollary 3 of Theorem 15.5
142.	See Theorem 15.6
143.	The degree of the zero polynomial is undefined
144.	This is true when the ring is a field
145.	See Theorem 16.1
146.	This is true when R is an integral domain
147.	If f and g have the same degree and their lead coefficients are additive inverses it is false
148.	See Theorem 16.3